C-Nav QA/QC Precision and Reliability Statistics

C-Nav World DGPS

730 East Kaliste Saloom Road
Lafayette, Louisiana, 70508
Phone: +1 337.261.0000
Fax: +1 337.261.0192

DOCUMENT CONTROL

Revision No	Author / reviser	Revision description / comments	Date	Approved
1.0	SA	Explanation of terms and concepts regarding C-Nav P^3QC precision measures and reliability statistics, and RAIM/RMS data format functionality.	06/16/09	
1.1	SA	Added $GPGST, $PNCTGST, $GPGBS and $PNCTMDE Data Output Message Format Tables. Added Data Presentation by Message Tables	07/31/09	TP

C-Nav P3QC Precision Measures (RMS & RAIM)

Exceeding both UKOOA and IMO regulations, C-Nav Receivers outfitted with P3QC firmware use statistically sound RAIM and RMS data formats to detect and reject outliers in acquired data for the output of precision GPS/GNSS measurements. Both present the same underlying data, but at different levels of precision.

The RMS data format represents measurement precision to one standard deviation, or one-sigma (1σ). Probability for RMS is 68.3%, meaning that if 100 observations are made, 68 will be within the root mean square, or 1 standard deviation. To put it in real-world terms, if a receiver claims an accuracy of 10cm at 1σ, then 68 out of 100 fixes will lie within a 10cm horizontal error ellipse, or in this case, a 68.3% confidence region.

The RAIM data format represents precision at the much more stringent 95% confidence level (2 standard deviations, or 2σ, as per UKOOA guidelines). In other words, if a receiver claims an accuracy of 20cm at the 95% confidence level, 95 out of 100 fixes will fall within a 20cm horizontal error ellipse, or, in this case, a 95% confidence region.

The RAIM data format also incorporates a statistical function called the F (Fisher)-test of Unit Variance, which is essentially a statistical test to determine whether the calculated unit variance is within the expected bounds as defined by the UKOOA guidelines. [1]

C-Nav P3QC Reliability Statistics (GBS & MDE)

Accuracy and precision are indicators/descriptors of the quality of measured data. In light of this, it is the function of the GBS and MDE data fields to represent alternative methods for ensuring the integrity/quality of the data and internal receiver processes (RAIM & RMS).

The purpose of the GBS data field is to provide statistical information about measurements that **are not** used in the solution by the RAIM algorithm. In other words, a probability is quoted for a given error, which means that the GBS field identifies the probability of a missed detection by the RAIM process. GBS data is required by certain IMO regulations and in statistical terms, represents a Type II Error Probability.

The purpose of the MDE data field is to act as a 'check' for the GBS processes, and thus represents an estimated error for a Type II Error Probability. MDE data is derived from measurements that **are** used in the solution once the RAIM process has excluded fixes deemed to be outliers. [2]

In short, within every P3QC-enabled C-Nav receiver lies the ability to detect outliers, reject those that fall outside acceptable UKOOA and IMO confidence levels, and then report pertinent information about all accepted and rejected measurements, allowing the user to achieve an unprecedented level of positioning confidence. For more information on the statistical value derivation processes and mathematics behind C-Nav P3QC enabled receivers, refer to:

Hedge, Tony. "Quality Statistics and the C-Nav 2050 GPS Receiver", available at www.cnavgps.com

[1] Hedge, Tony. "Quality Statistics and the C-Nav2050 GPS Receiver", July 2008
[2] Hedge, Tony. "A P3QC Statistics Glossary for the C-Monitor Help Files", January 2009.

Glossary of Terms

General Statistical Terms Used in GPS positioning [3]

Accuracy – The agreement of a value, whether measured or computed, with the standard or accepted true value. In the absolute sense, the true value is unknown and therefore, accuracy can only be estimated. Nevertheless, in measurement, accuracy is considered to be directly proportional to the attention given to the removal of systematic errors (bias) and mistakes. In GPS specifically, the values derived are usually the position, time, or velocity at GPS receivers.

Bias – A systematic error. Biases affect all GPS measurements, and hence the coordinates and baselines derived from them. Biases in GPS measurement may originate from imperfect satellite orbit data, atmospheric conditions, clock errors, etc. They may also originate from incorrect ephemeris information or less than perfect control coordinates. Modeling is one method used to eliminate, or at least limit the effect of biases.

Confidence Level – The probability that the true value is within a particular range of values, expressed as a percentage.

Confidence Region – A region, within which the true value is expected to fall, attended by a confidence level.

Integrity – A quality measure of GPS performance including a system to provide a warning when the system should not be used for navigation because of some inadequacy.

Outlier – Values that fall outside the expected or acceptable range from the mean in a given series of measurements.

Precision – Agreement among measurements of the same quantity. Widely scattered results are less precise than those that are closely grouped; the higher the precision, the smaller the random errors in a series of measurements.

RMS – The square root of the mean of squared errors for a sample. RMS is also referred to as the quadratic mean. It is a statistical measure of the magnitude of a varying quantity. It is a measure of the magnitude of a set of numbers.

Standard Deviation (one-sigma, 1σ) – An indication of the dispersion of random errors in a series of measurements of the same quantity. The more tightly grouped the measurements around their average (mean), the smaller the standard deviation. Approximately 68% of the individual measurements will be within the range expressed by the standard deviation.

Twice the Distance RMS – In practical terms, a particular 2DRMS value is the radius of a circle that is expected to contain from 95 to 98% of the positions a receiver collects in one occupation. In GPS, it can be predicted using covariance analysis by multiplying the HDOP by the standard deviation of the observed pseudoranges.

[3] Van Sickle, Jan. "GPS for Land Surveyors", CRC Press: 2nd Ed.

Data Presentation by Message [4]

C-Nav P[3]QC statistics are available via both NMEA and proprietary binary messages as follows:

Precision Measures

Precision Statistics	NMEA	Binary	Standards
Unit Variance	$GPGST $PNCTGST	B1	UKOOA UKOOA
F-test of Unit Variance	$PNCTGST	TBD	UKOOA
67% Error Ellipse (NMEA spec)	$GPGST	B5	NMEA
95% Error Ellipse (UKOOA spec)	$PNCTGST	TBD	UKOOA

Reliability Statistics

Reliability Statistics etc	NMEA	Binary	Standards
Outlier rejection (1 per rejected satellite, max 3)	$GPGBS	TBD	At least equivalent to UKOOA guidelines.
Type II error probability of most likely failed satellite remaining in solution (corresponding to β of w-test).	$GPGBS	TBD	NMEA/IMO
w-statistic (for each satellite remaining in solution)	$PNCTMDE	TBD	UKOOA spec, additional to guidelines.
Marginally detectable error in range and position (for each satellite remaining in solution) corresponding to α = 5%, β = 20%.	$PNCTMDE	TBD	UKOOA spec, exceeds guidelines specify only largest positional MDE.

[4] Hedge, Tony. "Quality Statistics and the C-Nav2050 GPS Receiver", July 2008.

Data Output Message Format Tables [5]

$GPGST

This output message reports pseudorange noise statistic information, and is in compliance with NMEA-0138 Standards version 3.0

Table 1: GST Message Output Format

Output Format:	$GPGST,time,rms,majoraxis,minoraxis,orientation,laterr,lonerr,alterr*checksum	
Field#	Field Name	Description
F1	time	UTC time for position fix in hours, minutes, seconds (hhmmss.ss) (000000.00 to 235959.99)
F2	rms	Total RMS standard deviation of ranges inputs to the navigation solution
F3	majoraxis	Standard deviation of semi-major axis of error ellipse in meters
F4	minoraxis	Standard deviation of semi-minor axis of error ellipse in meters
F5	orientation	Orientation of semi-major axis of error ellipse in true north degrees (0 to 180°)
F6	laterr	Standard deviation of latitude error in meters
F7	lonerr	Standard deviation of longitude error in meters
F8	alterr	Standard deviation of altitude error in meters
F9	*CRC	Checksum

Example:
*$GPGST,032746.00,22236.0738,0.0552,0.0355,019.4414,0.0543,0.0368,0.0991*6A*

$PNCTGST

This message satisfies the UKOOA compliance requirements by starting with the standard NMEA GST message and scaling all error statistics by 1.96, and by adding a value for the F-Test.

Table 2: PNCTGST Message Output Format

Output Format:	$PNCTGST,time,rms,majoraxis,minoraxis,orientation,laterr,lonerr,alterr,fisher*checksum	
Field#	Field Name	Description
F1	time	UTC time of the GGA or GNS fix associated with this sentence, represented as hours, minutes, seconds (hhmmss.ss) (000000.00 to 235959.99)
F2	rms	RMS value of the standard deviation of the range inputs to the navigation process. Range inputs include pseudoranges and DGNSS corrections.
F3	Majoraxis*	Standard deviation of semi-major axis of error ellipse in meters
F4	Minoraxis*	Standard deviation of semi-minor axis of error ellipse in meters
F5	Orientation	Orientation of semi-major axis of error ellipse in degrees from true north
F6	Laterr*	Standard deviation of latitude error in meters
F7	Lonerr*	Standard deviation of longitude error in meters
F8	Alterr*	Standard deviation of altitude error in meters
F9	Fisher	Fisher Test Result
F10	*CRC	Checksum

* Indicates the result is scaled by 1.96 This output stream reports pseudorange noise statistic information, and is in compliance with NMEA-0183 Standards version 3.0

Examples:
*$GNGST,192518.00,0.3762,0.1054,0.0953,074.8583,0.0960,0.1048,0.2168*7A*
*$PNCTGST,193028.00,0.2993,0.1722,0.1448,084.7181,0.1451,0.1720,0.3391,1*65*

[5] NavCom Technologies, "Sapphire GPS Engine - Technical Reference Manual", Draft G: July 2009.

$GPGBS

This output message is used to support Receiver Autonomous Integrity Monitoring (RAIM), and is in compliance with NMEA-0183 Standards version 3.0

***Table 1**: GBS Message Output Format*

Output Format:	$GPGBS,time,laterr,lonerr,alterr,prn,probability,estimate,bias*checksum	
Field#	Field Name	Description
F1	time	UTC time for position fix in hours, minutes, seconds (hhmmss.ss) (000000.00 to 235959.99)
F2	laterr	Expected error in latitude in meters
F3	lonerr	Expected error in longitude in meters
F4	alterr	Expected error in altitude in meters
F5	Prn	PRN of most likely failed satellite, GPS (01 to 32), WAAS (120-138)
F6	probability	Probability of missed detection for most likely failed satellite
F7	estimate	Estimate of bias in meters on most likely failed satellite
F8	bias	Standard deviation of bias estimate
F9		Checksum

Example:

*$GPGBS,233618.00,-0.2063,-0.0220,-0.4760,14,0.0001,-2.4018,8.5704*65*

$PNCTMDE

This output message reports the Marginally Detectable Error (MDE) generated by the receiver as part of the self-monitoring duties performed to support Receiver Autonomous Integrity Monitoring (RAIM). It is a proprietary NMEA type message, and it conforms to the header, checksum and electrical characteristics of a standard NMEA string, but is not recognized by the NMEA governing body as an officially sanctioned message.

***Table 2**: PNCTMDE Message Output Format*

Output Format:	$PNCTMDE,hhmmss.ss,s,t,b.b,MM,l.l,g.g,a.a*hh<CR><LF>	
Field#	Field Name	Description
F1	Time	UTC time for position fix in hours, minutes, seconds (hhmmss.ss) (000000.00 to 235959.99)
F2	svid	The GNSS svId
F3	Type	Measurement type: 0 = CA, 1 = P1, 2 = L1 , 3 = P2, 4 = L2, 5 = RC CODE, 6 = RC PHASE
F4	bias	standardized bias which is noncentrality parameter for w-test
F5	mde	MDE in meters
F6	laterr	Expected error in latitude (meters)
F7	longerr	Expected error in longitude (meters)
F8	alterr	Expected error in altitude (meters)
F9	*CRC	Checksum

Example:

*$PNCTMDE,165535.00,,,,,,*6A*

C-Nav P3QC RMS / RAIM (Precision Measures) GST/PNCTGST Data Field Definitions[6]

RMS / Unit Variance – The square of the calculated standard deviation for a data set with an expected mean value of 1 over time. In practice, a lower mean value (0.1 to 1.0) is more likely to indicate the absence of detrimental biases in GPS measurements.

F-Test (RAIM Only, PNCTGST Only) – The F-Test is a statistical test to determine whether the calculated unit variance is statistically within the expected bounds, as defined by the UKOOA guidelines. It is also present to allow the user to monitor the operation of the RAIM process.

Horizontal Error Ellipse – These 3 fields describe/define the horizontal error ellipse for a given set of measurements based on the 68 and 95% confidence levels, as per UKOOA guidelines.

Semi-Major Axis (m) – The length of the semi-major axis of the error ellipse at the applicable confidence level (1σ or 95%).

Semi-Minor Axis (m) – The length of the semi-minor axis of the error ellipse at the applicable confidence level (1σ or 95%).

Orientation (°) – The orientation (relative to true north) of the semi-major axis of the error ellipse.

Individual Coordinate Precision – These 3 fields represent the precision estimates of the individual coordinates for a given set of measurements based on the 68 and 95% confidence levels, as per UKOOA guidelines.

Latitudinal Precision (m) – Latitude precision estimate at the applicable confidence level (1σ or 95%).

Longitudinal Precision (m) – Longitude precision estimate at the applicable confidence level (1σ or 95%).

Height Precision (m) – Height precision estimate at the applicable confidence level (1σ or 95%).

[6] Hedge, Tony. "A P3QC Statistics Glossary for the C-Monitor Help Files", January 2009.

GBS / MDE (Reliability Statistics) Data Field Definitions[7]

Fields that include the (0x9d only) descriptor below are available only to C-Monitor or C-NaviGator CDU via the proprietary 0x9d binary QC message. These fields are in addition to fields available within the standard NMEA-0183 GBS data string.

Mode (0x9d only) – Indicates the correct mode of the solution from which the measurement is derived (Code, RTG, RTK or L1).

Status (0x9d only, not applicable to MDE) – Indicates the status of the GBS data for the satellite in question:

> **Invalid**: no GBS data has been generated for the satellite in the current epoch.
>
> **Used**: this GBS data represents a measurement, which has not been excluded from the solution by the RAIM process
>
> **Excluded**: the GBS data represents a measurement that has been identified as an outlier and is excluded from the navigation solution by the RAIM process.

Type (0x9d only) – Indicates the type of measurement associated with this data (C/A Pseudorange, P1 Pseudorange, L1 Carrier-Phase, P2 Pseudorange, L2 Carrier-Phase, P1/P2 Refraction Corrected Code, L1/L2 Refraction Corrected Carrier-Phase).

W-Test Statistic (MDE) – Also known as the RAIM non-centrality parameter. The w-statistic is one of the parameters recommended for computation by the UKOOA guidelines, in which a test of this statistic is used to reject outliers. The RAIM process uses an alternative (and more sensitive) technique for outlier rejection, but still computes a statistic that is mathematically equal to the w-test. This parameter is provided to enable the user to monitor the correct operation of the RAIM process.

Probability of Missed Detection (1.0 = 100%, GBS only) – For a measurement that has been excluded (GBS Status = 'Excluded'), this will always be zero. For a measurement that has not been excluded (GBS Status = 'Used'), this represents the probability that if this measurement was indeed an outlier, it could not be detected as such because the estimated measurement bias is so small.

This is a difficult and counter-intuitive concept – a high value here does not indicate a problem, it simply indicates that the measurement has a very small estimated bias, and so even if it was an outlier, it could not be distinguished from a valid measurement. Similarly, a small, non-zero value implies that although the estimated measurement bias was large, it was judged by the RAIM process not to be sufficiently large to warrant exclusion. In statistical testing terms, this is the Type II Error Probability associated with the estimated bias, and ranges from 0 to 0.95 – the probability is limited to 95% by the 5% value used for the Type I Error Probability (the probability of false rejection).

[7] Hedge, Tony. "A P3QC Statistics Glossary for the C-Monitor Help Files", January 2009.

Marginally Detectable Error (m) (MDE only) - Marginally Detectable Error, as per the UKOOA guidelines. It represents the bias in this measurement that would correspond to an 80% probability of detection (i.e. a Type II Error Probability of 20%). This does not imply that such a bias is present – it simply reflects the internal reliability of the solution, i.e. that such a bias would have a large chance of being detected.

Bias (m) and Noise (m) (GBS only) – Indicates the estimate of the bias for the measurement, i.e. the size of the estimated measurement error, computer after all outliers have been eliminated. In addition, a noise value is calculated that represents the standard deviation (noise estimate) associated with this measurement. A high value here indicates that the measurement in question would contribute little to the position solution.

Positional Errors - In GBS data (Bias) and in MDE data, (Marginally Detectable Error), both are estimates of a measurement (i.e. range) error. These can be transformed to an estimate of the impact on the position solution of including a measurement corresponding to the respective error.

> **Positional Errors (GBS)** - For a measurement that has been used (GBS Status = 'Used') it can be misleading to view this as an error. The RAIM process has used this measurement because it does not meet the criteria for rejection as an outlier. For a measurement that has been excluded as an outlier, these represent the errors that would have been caused by its use.
>
> **Positional Errors (MDE)** - This is the positional error that would occur if an error equal to the marginally detectable error were present in this measurement. It does not imply that such an error is present; it simply represents the external reliability of the solution.

North/East/Up Errors (m) (GBS only) - The components in the north, east and up directions of the effect on the position solution caused by (or, for excluded satellites, that would be caused by) including the measurement.

Latitude/Longitude/Height Errors (m) (MDE only) – The components in the latitude, longitude and height directions of the effect on the position solution that would be caused by including a measurement with a bias equal to the calculated MDE. This represents the external reliability of the solution and does not imply that such a bias is present.

References

Hedge, Tony. “A P3QC Statistics Glossary for the C-Monitor Help Files”, January 2009.

Hedge, Tony. “Quality Statistics and the C-Nav2050 GPS Receiver”, July 2008.

NavCom Technologies, “NavCom NMEA GBS/MDE/GST Message Software Design Document”, July 2008.

NavCom Technologies, “Sapphire GPS Engine - Technical Reference Manual”, Draft G: July 2009.

Van Sickle, Jan. “GPS for Land Surveyors”, CRC Press: 2nd Ed.